RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. FIRST SEMESTER EXAMINATION, DECEMBER 2013

FIRST YEAR

Mathematics (General)

Date : 17/12/2013

Time : 11am – 2pm

Paper : I

Full Marks : 75

(Use a separate answer book for each group)

Group – A

Answer **any five** questions of the following: (5 × 5)

1. State D' Moiver's theorem. Apply it to solve the equation $(x+1)^6 = (x-1)^6$. (1 + 4)

2. Prove that $\sin\left[i\log\frac{a-ib}{a+ib}\right] = \frac{2ab}{a^2+b^2}$. (5)

3. Show that the roots of the equation $\frac{1}{x-a}+\frac{1}{x-b}+\frac{1}{x-c}=\frac{1}{x}\,(a,b,c>0)$ are all real. (5)

4. a) State Rolle's theorem in connection to theory of equations. (1)

   b) If the equation $3x^4+4x^3-60x^2+96x-k=0$ has four real and unequal roots, show that 32<k<43. (4)

5. Solve the equation by Cardan's method:

   $x^3+3x^2-15x-52=0$. (5)

6. If ω is one of the imaginary cube roots of unity, prove that $a+b\omega+c\omega^2$ is a factor of $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$.

   Hence or otherwise show that $\Delta = -(a^3+b^3+c^3-3abc)$. (2 + 3)

7. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$. (5)

8. From the matrix equation $AX = B$, find X, Where $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix}$. (5)

Group – B

Answer **any five** questions of the following: (5 × 5)

9. a) Show that the following mapping f is injective but not subjective, where $f: N \to N$ defined by $f(n) = n+1, n \in N$. (2)

   b) Let $f: R \to R$ be defined by $f(x) = 3x+1, x \in R$. Prove that f^{-1} exists and find f^{-1}. (3)

10. a) If H, K are subgroups of a group (G, 0), then prove that $H \cap K$ is also a subgroup of (G, 0). (3)

    b) Give an example to show that the union of two subgroups of a group G is not necessarily a subgroup of G. (2)

11. a) Give an example of an abelian group. (1)

    b) Show that a non empty subset H of a group (G, 0) is a subgroup of (G, 0), if and only if

    i) $a, b \in H \Rightarrow aob \in H$ and

    ii) $a \in H \Rightarrow a^{-1} \in H$. (4)

12. a) Find the units in the ring $(Q, +, \cdot)$. (1)

    b) If R is a nontrivial ring with unity I, then show that $0 \neq I$. Furthermore give an example of a ring where 0 = I. (2 + 2)

13. Determine the eigen values of $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ and the eigen vectors corresponding to the positive eigen value of A. (3 +2)

14. Find the matrix associated with the real quadratic form: $x^2 + y^2 - z^2 + 2xy + 2zx - 2yz$. Find the eigen values of this matrix and hence find the nature of the real quadratic form. (5)

15. Verify whether S_1 and S_2 are subspaces of the real vector space R^3, where

 i) $S_1 = \{(x, y, z) \in R^3 : x - 3y + 4z = 0\}$

 ii) $S_2 = \{(x, y, z) \in R^3 : x^2 + y^2 \le z^2\}$. (5)

16. Use replacement theorem to find a basis of the real vector space R^3 containing the vectors (1, 1, 0) and (1, 1, 1). (5)

Group – C

Answer **any five** questions of the following : (5 × 5)

17. a) Find the domain of definition of the following function $f(x) = \log|4 - x^2|$. (2)

 b) Show by $\in -\partial$ definition of limit that $\lim_{x \to 3} \frac{1}{x+3} = \frac{1}{6}$. (3)

18. a) State Leibnitz's theorem on successive differentiation. (2)

 b) If $y = \cos(10\cos^{-1} x)$, show that $(1 - x^2)y_{12} = 21xy_{11}$. (3)

19. a) If $f(x) = 2|x| + |x - 2|$, test whether f is continuous at x = 2. Find $f^1(2)$, if possible. (4)

 b) Give an example of a real valued function which is nowhere differentiable. (1)

20. State Euler's theorem on homogeneous function. If $u = 2\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, apply Euler's theorem to prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \cot\frac{u}{2} = 0$. (2 + 3)

21. a) Verify that the double limit

$$\lim_{\substack{x \to 0 \\ y \to 0}} \frac{x+y}{x-y}$$

 does not exist. (2)

 b) Prove that the function $f(x, y) = \begin{cases} \frac{x^3 + y^3}{x - y}, & \text{when } x \ne y \\ 0, & \text{when } x = y \end{cases}$

 is not continuous at (0, 0). (3)

22. a) State the schwartz's theorem on commutative property of mixed second order derivatives. (2)

 b) If u be a homogeneous function of x and y of degree n, then show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

 $\left(\text{assume}: \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}\right)$ (3)

23. Find the radius of curvature at the point (r, θ) on the cardiode $r = a(1 - \cos\theta)$ and show that it varies as $\sqrt{r}$. (4 + 1)

24. Show that the pedal equation of the parabola $y^2 = 4a(x + a)$ with respect to the origin is $p^2 = ar$. (symbols have their usual meaning). (5)